Growth of a driven interface in isotropic and anisotropic random media

Kwangho Park, Hyun-Joo Kim, and In-mook Kim
Department of Physics, Korea University, Seoul 136-701, Korea
(Received 12 May 2000)

We introduce a simple stochastic model for a driven interface in a random medium, in which we can control the degree of the anisotropy of a random medium. When there is no anisotropy of a random medium, the motion of a growing interface in our model can be well described by the quenched Edwards-Wilkinson equation. When there is anisotropy of a random medium, however, the motion of a growing interface can be described by the quenched Kardar-Parisi-Zhang (KPZ) equation. In the two interfaces, apart from one growing in an isotropic medium and the other growing in an anisotropic medium, the growth rule of our model is the same. Our results support the fact that the anisotropy of a random medium is a source of the KPZ nonlinearity.

PACS number(s): 05.40.−a, 68.35.Fx, 47.55.Mh

Depinning of a driven interface in a random medium has been a popular research topic for a long time, because it frequently occurs, e.g., in random magnets [1], in fluid invasions in porous media [2], and in depinning charge-density waves [3]. There have been many studies about the depinning of a driven interface in a random medium via stochastic growth models [4–10], renormalization-group analysis [11,12], continuum equations [13], various experiments [14,15], etc.

Depinning dynamics of a driven interface can be well explained by a simple Langevin-type continuum equation [13],

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}(\nabla h)^2 + \eta(x,h) + F, \tag{1}$$

where $h(x,t)$ is the height of the interface at position x at time t. The quenched noise satisfies $\langle \eta(x,h)\rangle = 0$ and $\langle \eta(x,h)\eta(x',h')\rangle = \delta^{d'}(x-x')\delta(h-h')$. Here d' means the substrate dimension. When $\lambda \neq 0$, Eq. (1) is called the quenched Kardar-Parisi-Zhang (QKPZ) equation [13]. When $\lambda = 0$, Eq. (1) is called the quenched Edwards-Wilkinson (QEW) equation [13]. The interface in Eq. (1) is pinned when the driving force F is smaller than F_c. However, the interface moves with a constant velocity $v \sim (F-F_c)^{\theta}$ for $F > F_c$, where θ is the velocity exponent. This phenomenon is called the pinning-depinning (PD) transition.

Near the depinning threshold, the depinned interface shows a nontrivial scaling behavior in the global interface width,

$$W(L,t) \equiv \left\langle \frac{1}{L^{d'}} \sum_x [h(x,t) - \bar{h}(t)]^2 \right\rangle^{1/2}, \tag{2}$$

where $\bar{h}(t)$ and L denote the mean height at time t and system size, respectively. The symbol $\langle \cdots \rangle$ stands for the statistical average. The surface width shows a scaling behavior $W \sim L^{\zeta} f(t/L^z)$, where the scaling function $f(x)$ approaches a constant for $x \gg 1$, and $f(x) \sim x^{\beta}$ for $x \ll 1$ with $z = \zeta/\beta$ [13]. The exponents ζ, β, and z are called the roughness, the growth, and the dynamic exponent, respectively. The roughness exponent is known to be $\zeta \simeq 0.63$ for the QKPZ equation and $\zeta \simeq 1 \sim 1.25$ for the QEW equation in $d' = 1$ [13]. Analytical and numerical (using the model corresponding to the discretization of the QEW equation) studies of the QEW equation give us roughness exponents $\zeta \simeq 1$ and $\zeta \simeq 1.25$, respectively [11–13]. Also the roughness exponents of various models, which are accepted to belong to the QEW universality class, are scattered between $\zeta \simeq 1$ and $\zeta \simeq 1.25$ [7–10].

Many stochastic models [4–10], which mimic the motion of the driven interface in a random medium near the depinning threshold, were introduced and studied. Amaral *et al.* [16] observed that the numerical results obtained in the study of the stochastic models fall into two groups. In one group, the growing velocity $v(s)$ of a driven interface does not depend on the slope s of a tilted substrate near the depinning threshold or becomes independent of s at the depinning threshold, although there is the dependence of $v(s)$ on s far from the depinning threshold. The stochastic models in this group are known to belong to the QEW universality class. In the other, $v(s)$ depends on s even at the depinning threshold. The stochastic models in this group are known to belong to the QKPZ universality class. In addition to the study of Amaral *et al.*, Tang *et al.* [17] argued that the critical behavior of a driven interface in a random medium at the depinning threshold depends on whether the random medium is isotropic or anisotropic. When the random medium is isotropic, $v(s)$ does not depend on slope s. When the random medium is anisotropic, however, $v(s)$ depends on slope s. They also suggested a method to find out whether the random medium is isotropic or anisotropic. The method measures the dependence of the depinning threshold force $F_c(s)$ on the slope s. They argued that the dependence of $F_c(s)$ on slope s originates from the anisotropy of the medium. By carrying out stochastic model simulations, they showed that $F_c(s)$ depends on s in models where $v(s)$ depends on s at the depinning threshold.

In this paper, we introduce a simple stochastic growth model for a driven interface in a random medium near the depinning threshold, where we can control the degree of anisotropy of the medium. From the study, we show that the dependence of $v_c(s)$ on s indeed originates from the anisotropy of the medium.

We consider a simple self-organized automaton model (SOAM), which was originally introduced by Leschhorn [18] to mimic the motion of a driven interface in an isotropic medium at the depinning threshold. We modified a bit the growth rule of the original SOAM in order to control the degree of the anisotropy of the medium. The growth rule of our model is as follows. (i) We assign a random number between 0 and 1 on each lattice site in a $(1+1)$-dimensional system, where the random number represents the impurities of a random medium. (ii) For each time t we calculate the local force for all site i,

$$f_i(t)=\sum_{\langle j\rangle}[h_j(t)-h_i(t)]+m(1+g*\tilde{s}_i)\eta_{i,h_i}, \tag{3}$$

where the sum is over the nearest neighbors of site i, i.e., $j=i\pm1$, and m and g are integers. $h_i(t)$ denotes the height at time t and site i. η_{i,h_i} denotes the random number at site i and height h_i. The local slope $\tilde{s}_i$ is zero only when $h_j-h_i=0$, otherwise $\tilde{s}_i=1$. (iii) We increase the column having the maximum $f_{\max}\equiv\max[f_i]$ among all f_i as follows:

$$h_i(t+1)=h_i(t)+1 \quad \text{if} \quad f_i=f_{\max},$$
$$h_i(t+1)=h_i(t) \quad \text{otherwise}. \tag{4}$$

When g is 0, the growth rule of the model is the same as that of the original SOAM. Dynamic behavior of the original SOAM is known to be well described by the quenched Edwards-Wilkinson (QEW) equation near the depinning threshold. Leschhorn [18] obtained the roughness exponent ζ by doing a computer simulation of the original SOAM. The obtained roughness exponent is $\zeta=1.24\pm0.01$ in $1+1$ dimensions. By carrying out computer simulations on our model when $m=2$ and $g=0$, we obtained $\zeta\simeq1.25$ (Fig. 2). We found through some simulations for different values of m that the value of the roughness exponent does not depend on the value of m. Therefore, we used a fixed value of $m=2$ in our simulation.

As the original SOAM is well described by the QEW equation, our model can be described by the following continuum equation with $F=F_c$:

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2h(x,t)+\tilde{\eta}(x,h)+F, \tag{5}$$

where the quenched noise satisfies the condition $\langle\tilde{\eta}(x,h)\rangle=0$ and $\langle\tilde{\eta}(x,h)\tilde{\eta}(x',h')\rangle=[1+f(\tilde{s})]\delta(x-x')\delta(h-h')$. $f(\tilde{s})$ is a function depending on local slope $\tilde{s}=|\nabla h|$. When $\tilde{s}$ is nonzero, $f(\tilde{s})\neq0$. A simple scaling argument suggests that there is an important length scale in Eq. (5). If the length in Eq. (5) is known, the critical driving force F_c in Eq. (5) can be determined. Let us denote by l the domain size which blocks the motion of the interface in a random medium when a driving force pushes the interface. One can rewrite Eq. (5) for the l region as follows:

$$l^d\nu l^{-2}h+l^dF-[1+f(\tilde{s})]^{1/2}l^{d/2}=0, \tag{6}$$

where the first term comes from $\nu\nabla^2h$, the second is the driving force, and the last is the contribution from the noise, which has a negative sign because we assume that it opposes the motion of the interface. When the driving force F is zero, the interface is always pinned and there is no characteristic length scale. When the driving force is nonzero, however, there can exist the characteristic length scale. The interface becomes flat if the Laplacian term is stronger than the pinning force by the noise. In that case, the interface is in a depinned state if the driving force F is nonzero [19]. In $d>d_c=4$ dimensions, the Laplacian term will always win for a very weak pinning force, and there is no pinning of the interface. Therefore, there is no characteristic length scale for $d>d_c$. The situation is different for $d<d_c$. The equation $a_\perp\nu l^{d-2}\ll[1+f(\tilde{s})]^{1/2}l^{d/2}$ provides the characteristic length scale

$$l_c\sim\left(\frac{\nu^2a_\perp^2}{[1+f(\tilde{s})]^{1/2}}\right)^{1/(4-d)}, \tag{7}$$

where $a_\perp$ is a constant. For $\epsilon=4-d>0$, the smoothening effect of $\nu\nabla^2h$ dominates for the length scale $l\ll l_c$, while for $l\gg l_c$ the interface wanders, taking advantage of the low-energy configuration in the disorder. In the case of $l\gg l_c$, the pinning force term wins the Laplacian term. Therefore, the driving force needs to be larger than the maximum pinning force in order for the interface to be in a depinned state. The maximum pinning force F_c can be obtained from Eq. (6) by equating the driving force with the pinning effect of the noise and using Eq. (7),

$$F_c(s)\simeq\{[1+f(\tilde{s})]/l_c^d\}^{1/2}\sim[1+f(\tilde{s})]^{2/\epsilon}. \tag{8}$$

F_c is the critical driving force of the pinning-depinning transition. When g is nonzero, there exists the dependence of $F_c(s)$ on slope s, although we do not know the exact functional form of $F_c(s)$. Therefore, by putting nonzero g in our model, we can control the anisotropic effect of a random medium.

We carried out computer simulation of our model for $g=0$, 0.5, 1, 5, and 10. Numerical data were averaged typically over 100 configurations. In order to obtain the growth exponent, we measured the time-dependent behavior of the interface width $W(L,t)$ starting from the initially flat interface. We plot $W^2(L,t)$ versus time t in double-logarithmic scale in Fig. 1. The interface width grows with the exponent $\beta\simeq0.73$ for $g=0$. However, we could not obtain the value of the growth exponent for larger values of g because the width saturates as soon as the interface starts growing. The value of the obtained growth exponent at $g=0$ is a bit smaller than that obtained from some stochastic growth models in the QEW universality class, but is in comparatively good agreement with that value, 0.75, expected from the analytical solution of the QEW equation [11,12].

In order to obtain the roughness exponent, we plot the saturated value of $W^2(L,t)$ versus system size L in double logarithmic scale in Fig. 2. We obtained $\zeta=1.25$ in the QEW universality class when $g=0$ and $\zeta\simeq0.65$ for large values of g $(\geqslant5)$ in the QKPZ universality class. For small values of g $(0.5\leqslant g\leqslant1)$, we could not obtain the exact roughness exponent because of the crossover behavior. However, it is possible to conclude that the local slope of the width in Fig. 2 decreases as the system size becomes larger. In view of the result when $g\geqslant5$ and the decrease of the local slope in large

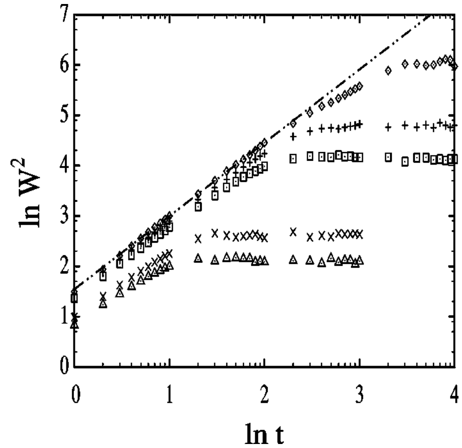

FIG. 1. The plot of width $W^2(t)$ vs t is shown for $g=0$, 0.5, 1, 5, and 10 from top to bottom, respectively. The system size L is 4096. The straight line represents $2\beta=1.46$.

system size when $0<g<1$, we argue that our model has $\zeta \simeq 0.65$ when $g>0$ and belongs to the QKPZ universality class.

We have also measured the height-height correlation function $C(x)$ defined as

$$C(x)=\left\langle \frac{1}{L^{d'}} \sum_{x} [h(x+x_1,\tau)-h(x_1,\tau)]^2 \right\rangle^{1/2}, \quad (9)$$

where time τ is larger than the saturation time, and $C(x)$ scales as $x^{\zeta'}$. The roughness exponent value from $C(x)$ is $\zeta' \simeq 0.96$ when $g=0$ and $\zeta' \simeq 0.64$ when $g \geqslant 1$ (see Fig. 3). The values of ζ' when $g=0$ and when $g \geqslant 1$ agree well with those expected from the QEW and the QKPZ universality class, respectively. When $g=0$, the value of ζ' is smaller than the one obtained from the interface width. It is well known that the anomalous scaling of the local width is due to the super-roughening, in such a way that the roughness exponent ζ' obtained from the height-height correlation function is smaller than the one obtained from the saturated value of $W^2(L,t)$ [7–9]. Super-rough scaling occurs when the roughness exponent of the width is $\zeta>1$. When $g=1$ and

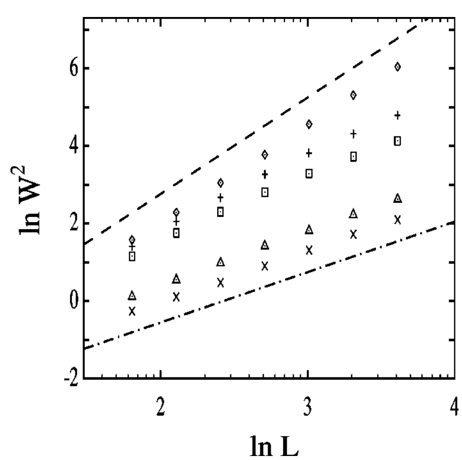

FIG. 2. The plots of width $W^2(L)$ vs L is shown for $g=0$, 0.5, 1, 5, and 10 from top to bottom, respectively. The system size is $L=64$–4096. The top line represents $2\zeta=2.5$ and the bottom line represents $2\zeta=1.3$.

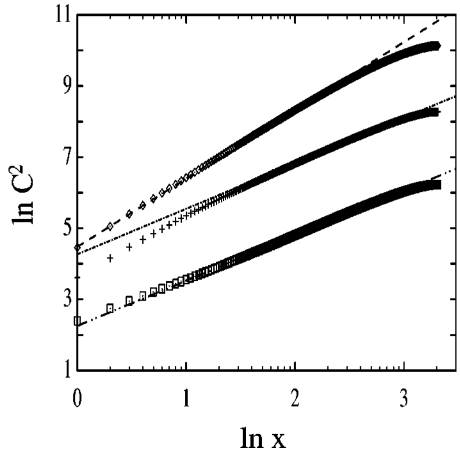

FIG. 3. The plot of the height-height correlation function $C^2(x)$ vs x is shown for $g=0$ (top), $g=1$ (middle), and $g=10$ (bottom) with the system size $L=4096$. The top straight line represents $2\zeta'=1.92$. The middle straight line represents $2\zeta'=1.28$. The bottom straight line represents $2\zeta'=1.27$.

$L=4096$, the roughness exponent ζ' shows a crossover behavior from $\zeta'>0.64$ for small values of x to $\zeta' \simeq 0.64$ for large values of x. These results also support the fact that our model has $\zeta' \simeq 0.64$ when $g>0$ and so belongs to the QKPZ universality class.

Several years ago, Wolf studied the anisotropic KPZ (AKPZ) equation with annealed noise in $2+1$ dimensions by using the dynamic renormalization-group (RG) method [20]. The annealed-noise AKPZ equation is written as

$$\frac{\partial h}{\partial t}=\nu_\perp \nabla_\perp^2 h(x,t)+\nu_\parallel \nabla_\parallel^2 h(x,t)+\frac{\lambda_\perp}{2}(\nabla_\perp h)^2 + \frac{\lambda_\parallel}{2}(\nabla_\parallel h)^2+\eta(x,t), \quad (10)$$

where $\nabla_\perp$ ($\nabla_\parallel$) is the gradient along the perpendicular (parallel) directions. The annealed noise satisfies $\langle \eta(x,t)\rangle=0$ and $\langle \eta(x,t)\,\eta(x',t')\rangle=\delta^{d'}(x-x')\,\delta(t-t')$. The anisotropy means $\nu_\parallel/\nu_\perp \neq 1$ and $\lambda_\parallel/\lambda_\perp \neq 1$. He found that when the signs of λ's are opposite, the nonlinear terms turn out to be irrelevant under the RG transformation. Therefore, the annealed-noise AKPZ equation with opposite signs of λ belongs to the weak-coupling limit, the annealed EW universality class. However, the crossover process from the QKPZ to the QEW universality class in our model occurs by a different mechanism from that occurring in the annealed-noise AKPZ equation. In our model, the KPZ nonlinearity is induced by an anisotropic random medium, and the nolinearity disappears if the medium is isotropic. This is because there is no source of producing the KPZ nonlinearity in the growth rule of our model except the anisotropy of the medium.

In summary, we have introduced a simple growth model for a driven interface in a random medium, where the degree of anisotropy of the medium is controlled by a parameter g. At $g=0$, there is no anisotropy of the medium in our model. For $g>0$, however, there exists anisotropy of the medium in our model. By carrying out the Monte Carlo simulation of our model, we found that our model belongs to the quenched Edwards-Wilkinson universality class when $g=0$. We then found that our model belongs to the quenched Kardar-Parisi-Zhang universality class when $g>0$. These facts support the argument that the anisotropy of the medium can produce the KPZ nonlinearity in the interface driven through the random media near the depinning threshold. It is well known that the KPZ nonlinearity induces the dependence of $v_c(s)$ on slope s of the driven interface in a random medium. From the model simulation, we showed that the dependence of $v_c(s)$ on s indeed originates from the anisotropy of the medium.

This work was supported in part by the Korean Science and Engineering Foundation (Contract No. 98-0702-05-01-3) and also in part by the Ministry of Education through the BK21 Project.

[1] M. Cieplak and M.O. Robbins, Phys. Rev. Lett. **60**, 2042 (1998).
[2] M. Sahimi, *Flow and Transport in Porous Media and Fractured Rock* (VCH, Weinheim, 1995).
[3] O. Narayan and D.S. Fisher, Phys. Rev. B **46**, 11 520 (1992).
[4] S.V. Buldyrev, A.-L. Barabási, F. Caserta, S. Havlin, H.E. Stanley, and T. Vicsek, Phys. Rev. A **45**, R8313 (1992).
[5] L.-H. Tang and H. Leschhorn, Phys. Rev. A **45**, R8309 (1992).
[6] K. Sneppen, Phys. Rev. Lett. **69**, 3539 (1992).
[7] S. Roux and A. Hansen, J. Phys. I **4**, 515 (1994).
[8] H.J. Jensen, J. Phys. A **28**, 1861 (1995).
[9] H.A. Makse and L.A.N. Amaral, Europhys. Lett. **31**, 379 (1995).
[10] K. Park and I.-m. Kim, Phys. Rev. E **59**, 5150 (1999); **61**, 4606 (2000).
[11] T. Nattermann, S. Stepanow, L.-H. Tang, and H. Leschhorn, J. Phys. II **2**, 1483 (1992).
[12] O. Narayan and D.S. Fisher, Phys. Rev. B **48**, 7030 (1993).
[13] A.L. Barábasi and H.E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, 1995).
[14] V.K. Horváth and H.E. Stanley, Phys. Rev. E **52**, 5166 (1995).
[15] M.A. Rubio *et al.*, Phys. Rev. Lett. **63**, 1685 (1989); V.K. Horváth, F. Family, and T. Vicsek, J. Phys. A **24**, L25 (1991); S. He *et al.*, Phys. Rev. Lett. **69**, 3731 (1992).
[16] L.A.N. Amaral, A.-L. Barabási, and H.E. Stanley, Phys. Rev. Lett. **73**, 62 (1994).
[17] L.-H. Tang, M. Kardar, and D. Dhar, Phys. Rev. Lett. **74**, 920 (1995).
[18] H. Leschhorn. Ph. D. thesis, Ruhr University, Bochum, 1994.
[19] A.I. Larkin, Zh. Éksp. Teor. Fiz. **58**, 1466 (1970) [Sov. Phys. JETP **31**, 784 (1970)].
[20] D.E. Wolf, Phys. Rev. Lett. **67**, 1783 (1991).